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A weighted least squares method for inverse dynamic analysis*

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Internal forces in the human body can be estimated from measured movements and external forces using inverse dynamic analysis. Here we present a general method of analysis which makes optimal use of all available data, and allows the use of inverse dynamic analysis in cases where external force data is incomplete. The method was evaluated for the analysis of running on a partially instrumented treadmill. It was found that results correlate well with those of a conventional analysis where all external forces are known.

Keywords: Inverse dynamics; Gait analysis; Multibody dynamics; Ground reaction force

1. Introduction

The recursive Newton–Euler method is widely used for inverse dynamic analysis of human movement (Winter 1979, Vaughan *et al.* 1992, van den Bogert 1994). These methods are applicable to multibody systems with a tree structure, with rigid body equations of motion being applied sequentially to each body segment, starting at distal segments where external loads are either zero or measured. The result is a full set of intersegmental load variables, i.e. a force and moment vector at each joint. This method is fast and easily implemented but has some undesirable properties. First, the results are dependent on the order in which the model is traversed. In lower extremity studies, the analysis is typically started at the feet, working towards the pelvis (Winter 1979). For the upper extremity, the analysis starts at the hands, working towards the shoulder (Fleisig *et al.* 1995). When estimating forces in the spine, it is not clear which of the two starting points is best (de Looze *et al.* 1992). Second, when the analysis is carried out for the entire human body, "residual loads" are needed at the final segment to satisfy the equations of motion, even when it is known that the final segment does not have contact with the environment. Kuo (1998) recognized that these shortcomings arise from the fact that the system of equations is overdetermined. For instance, if a 3D linked segment model has N-degrees of freedom (DOF), and all external forces are known or measured, there are N equations of motion and only $N-6$ unknown internal loads. The conventional method effectively solves this by discarding six of the equations, and the results will then depend on which six equations are eliminated. Furthermore, all kinematic and force measurements that entered in those six equations remain unused, even if they contain potentially useful information.

Kuo (1998) proposed an alternative method which solves joint moments from the overdetermined system of motion equations for the entire system, while satisfying the boundary conditions for a postural control task. The method finds a set of joint moments that best agrees (in the least squares sense) with all available measurements of kinematics and external forces. Redundancy in the system of equations is attractive when certain measurements are unreliable, or even unavailable such as in instrumented treadmills with only vertical force transducers. With complete data, Kuo (1998) demonstrated about a 30% noise reduction when compared to the conventional recursive analysis. The method was applicable only to a 2D system jointed to the ground, and was therefore, not suitable for gait analysis.

Here we present a further development of this least squares inverse dynamics (LSID) method that is no longer restricted to 2D systems jointed to ground. The method

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was implemented as a general software tool that allows arbitrary 3D or 2D models to be defined using markers on the subject and generates and solves the kinematic and dynamic equations automatically. The method can produce an optimal solution for any inverse dynamics problem as long as the number of unknown load variables does not exceed the number of DOF. The utility of the method will be demonstrated on an analysis of running with incomplete ground reaction force (GRF) data.

2. General methodology

2.1 Kinematic analysis

First, a skeleton model is defined with N DOF and generalized coordinates $\mathbf{q} = (q_1 \ldots q_N)^{\mathrm{T}}$. Assuming known joint axes, the position $\mathbf{r} = (x, y, z)^{\mathrm{T}}$ and orientation $\mathbf{R}$ of each body segment can be computed using forward kinematics as functions of $\mathbf{q}$. If a marker i is placed at a known position $\mathbf{p}$ in the segment's reference frame, the global coordinates $\mathbf{r}_i$ of the marker are therefore a known function of $\mathbf{q}$:

$$\mathbf{r}_i = \mathbf{r}(\mathbf{q}) + \mathbf{R}(\mathbf{q}) \cdot \mathbf{p}_i \equiv \mathbf{f}_i(\mathbf{q}) \tag{1}$$

If M markers are placed on the skeleton and their global 3D coordinates are measured, the optimal (least squares) estimate for the skeleton pose $\mathbf{q}$ can be obtained by minimizing

$$\mathbf{F}(\mathbf{q}) = \sum_{i=1}^{M} \|\mathbf{r}_i - \mathbf{f}_i(\mathbf{q})\|^2 \tag{2}$$

Note that in 3Ds, the right hand side is a sum of $3M$ squares. A unique minimum exists if $3M \geq N$ and $3m \times N$ the Jacobian matrix $\mathbf{J} = \partial \mathbf{f}/\partial \mathbf{q}$ is non-singular. This method is commonly referred to as *global optimization* (Lu and O'Connor 1999), where *global* refers to the fact that the entire skeleton is modeled, rather than isolated bones as in conventional rigid body motion analysis (Challis 1995). If an appropriate skeleton model and marker set is used, global optimization requires fewer markers and produces more robust results than conventional rigid body methods (Lu and O'Connor 1999, Roux *et al.* 2002).

In our implementation (Mocap Solver 6.19, Motion Analysis Corp., Santa Rosa, CA), equation (2) is minimized using the LMDIF code for nonlinear least squares problems (Moré *et al.* 1980) which is available from MINPACK at http://www.netlib.org. We also have obtained good results with the Levenberg–Marquardt solver in Numerical Recipes (Press *et al.* 1992) which is faster but somewhat less robust in situations where $\mathbf{J}$ is near-singular.

2.2 Dynamic analysis

After adding mass properties to the skeleton model, its equations of motion can be derived as:

$$\mathbf{M} \cdot \ddot{\mathbf{q}} = \mathbf{A}(\mathbf{q}) \cdot \boldsymbol{\tau}_{\mathbf{u}} + \mathbf{B}(\mathbf{q}) \cdot \boldsymbol{\tau}_{\mathbf{k}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}), \tag{3}$$

where $\mathbf{M}$ is a mass matrix, $\boldsymbol{\tau}_{\mathbf{u}}$ is a vector of unknown forces and moments, $\boldsymbol{\tau}_{\mathbf{k}}$ is a vector of known forces and moments, and $\mathbf{c}$ are the gravitational, centrifugal and Coriolis effects. $\mathbf{A}$ and $\mathbf{B}$ are coefficient matrices. After using spline smoothing (Woltring 1986) to obtain first and second derivatives of $\mathbf{q}(t)$, the only remaining unknowns are $\boldsymbol{\tau}_{\mathbf{u}}$. In order to avoid inconsistency in frequency content between the force and motion measurements, force measurements $\boldsymbol{\tau}_{\mathbf{k}}$ are smoothed with the same spline filter (van den Bogert and de Koning 1996, Bisseling and Hof 2006). If, as is typically the case in whole body models, the number of unknown forces and moments in $\boldsymbol{\tau}_{\mathbf{u}}$ is less than the number of equations (number of DOF) N, the system of equations is overdetermined and a linear least squares method can be used. Unlike in the kinematic analysis, weighting is required because here the N equations may have different error levels, scaling relationships, or units of measurement. We first rewrite (3) as:

$$\mathbf{A} \cdot \boldsymbol{\tau}_{\mathbf{u}} = \mathbf{b} + \mathbf{e} \tag{4}$$

where

$$\mathbf{b} = \mathbf{B}(\mathbf{q}) \cdot \boldsymbol{\tau}_{\mathbf{k}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{M} \cdot \ddot{\mathbf{q}} \tag{5}$$

and $\mathbf{e}$ is a vector of residual errors. The weighed least squares solution is:

$$\boldsymbol{\tau}_{\mathbf{u}} = \arg \min_{\boldsymbol{\tau}_{\mathbf{u}}} \left[(\mathbf{A} \cdot \boldsymbol{\tau}_{\mathbf{u}} - \mathbf{b})^{\mathrm{T}} \mathbf{W} (\mathbf{A} \cdot \boldsymbol{\tau}_{\mathbf{u}} - \mathbf{b}) \right], \tag{6}$$

where the weighting matrix $\mathbf{W}$ is the inverse of the covariance matrix of the error vector $\mathbf{e}$.

In order to find the covariance matrix, we consider that the noise in $\mathbf{q}(t)$ is small, while the error in first and second derivatives can be substantial even after optimal smoothing (Woltring 1985). We therefore assume that the matrix $\mathbf{A}$ does not contribute to $\mathbf{e}$, and we only consider the error in $\mathbf{b}$ which is the result of the propagation of measuring errors in $\boldsymbol{\tau}_{\mathbf{k}}$, $\mathbf{q}$, $\dot{\mathbf{q}}$, and $\ddot{\mathbf{q}}$. There are strong correlations between the elements of $\mathbf{b}$, because of the coefficient matrices and because of the whole body kinematic solution in which each marker coordinate contributes to each generalized coordinate. We can, therefore, not assume that the covariance matrix is diagonal. An analytical derivation would be intractable, so we use Monte Carlo simulation to estimate the covariance matrix from errors in raw data. The raw data are the marker coordinates $\mathbf{r}_i$ and the force measurements on which $\boldsymbol{\tau}_{\mathbf{k}}$ depends. We assume normally distributed errors σ_r (mm) in each marker coordinate, and normally distributed errors σ_τ (N or Nm) in measured force and moment variables. We take one typical recording of the motion of interest, perturb each sample of raw data with normally distributed random numbers with standard deviations σ_r and σ_τ, and propagate the data through the kinematic analysis, spline smoothing, and finally through equation (5). This is done a number of times on the same motion data to obtain a large number of perturbed vectors

b from which the covariance matrix **COV** is then estimated as:

$$\mathbf{COV}_{ij} = \frac{1}{N_p N_f} \sum_{k=1}^{N_f} \sum_{l=1}^{N_p} (b_{ikl} - b_{ik})\left(b_{jkl} - b_{jk}\right) \qquad (7)$$

where b_{ik} is the unperturbed ith element of **b** in sample k, and b_{ikl} is the kth perturbation of this variable. N_f is the number of frames (samples), and N_p is the number of perturbations applied to each frame. We use $N_p = 50$.

Once the covariance matrix is known, we compute its square root **S** using Cholesky factorization, such that $\mathbf{S}{\cdot}\mathbf{S}^{\mathbf{T}} = \mathbf{COV}$. Equation (6) is now equivalent to:

$$\boldsymbol{\tau}_{\mathbf{u}} = \arg\min_{\boldsymbol{\tau}_{\mathbf{u}}} \left\|\mathbf{S}^{-1}(\mathbf{A}{\cdot}\boldsymbol{\tau}_{\mathbf{u}} - \mathbf{b})\right\|, \qquad (8)$$

Equation (7) was solved using the DGGGLM General Linear Regression solver which is available from the LAPACK library at http://www.netlib.org. DGGGLM is based on QR decomposition of the matrices **A** and **S** (Golub and Van Loan 1989).

2.3 Equations of motion

There are many ways to derive equations of motion in the form (3). We used the SD/Fast software (PTC, Needham, MA) to generate the equations of motion. SD/Fast produces a triangular mass matrix which is advantageous for forward dynamics but has no particular advantage here because the mass matrix is never inverted. We obtain the mass matrix **M** using the SD/Fast function SDMASSMAT. SD/Fast also has a function SDFRCMAT which computes the right hand side of (3) for given kinematic state $(\mathbf{q}, \dot{\mathbf{q}})$ and applied forces. The latter function was used to obtain first the column vector $\mathbf{c}(\mathbf{q}, \dot{\mathbf{q}})$, by setting all forces to zero, and then to obtain the columns of matrices **A** and **B** by successively applying a unit force in each component of $\boldsymbol{\tau}_{\mathbf{u}}$ or $\boldsymbol{\tau}_{\mathbf{k}}$, while keeping all other components zero. A new **M**, **A**, **B** and **c** is thus computed in each sample of the movement. A more general symbolic manipulation method, such as Autolev (Online Dynamics, Sunnyvale, CA), would be able to extract **A**, **B** and **c** directly which would be more efficient.

3. Example of application

3.1 Problem statement

We will consider the inverse dynamic analysis of a running movement. Three dimensional joint moments are thought to be relevant to injury prevention and rehabilitation (Ferber *et al.* 2003). In the conventional gait laboratory, with a force platform in the ground, it is not possible to collect the required data continuously while the patient runs at their mechanical and metabolic steady state. Treadmill running is therefore an attractive paradigm but this does not allow full 6-component GRF (3D force and moment) to be recorded, which is required for conventional recursive inverse dynamic analysis when starting at the feet. A relatively inexpensive option is an instrumented treadmill with a force platform under the belt (GaitWay, Kistler, Amherst NY). This instrumentation only measures three of the 6 external load variables: vertical force and center of pressure. There is, however, currently no method for inverse dynamic analysis that can use such partial instrumentation. This inverse dynamic problem, however, fits nicely into the least squares framework presented above. With an N-DOF linked segment model in 3Ds, there will be $N - 6$ unknown internal loads, 3 unknown external loads, and n equations of motion. The number of equations (n) exceeds the number of unknowns ($n - 3$). We will demonstrate the utility of the least squares method on this problem.

3.2 Instrumentation and protocol

Twenty-eight reflective markers were placed on a 44 year old male subject (figure 1). Markers were tracked with six Falcon cameras (Motion Analysis Corp., Santa Rosa, CA) and EVa 5.2 software at 240 frames per second. GRF data were collected with an AMTI force plate (OR6-5 #4048, Advanced Mechanical Technology Inc., Watertown, MA) at 1000 samples per second. Data were collected during standing, followed by 23 trials of running at the subject's preferred speed. The subject was instructed to vary running style between trials, in order to test the ability of the inverse dynamic analysis to detect these variations.

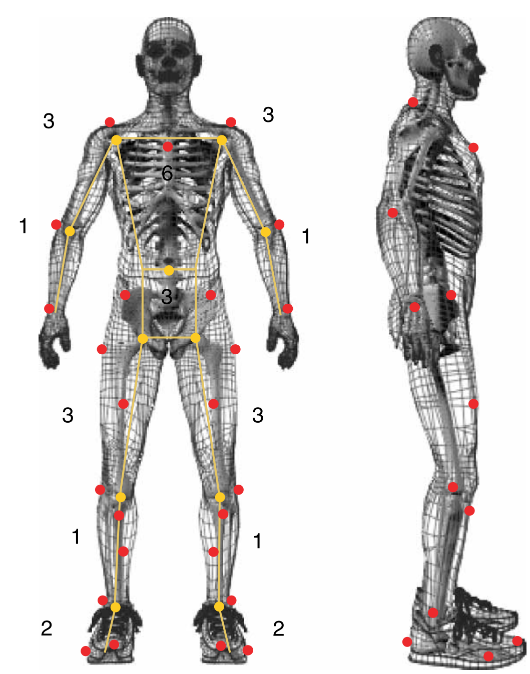

Figure 1. Marker set and skeleton model. Numbers indicate the number of DOF assigned to each of the body segments.

3.3 Data processing

From the marker coordinates during standing, a twelve-segment skeleton model (figure 1) was generated with 29 DOF: six for trunk position and orientation, three for a spherical joint between pelvis and trunk, three for each spherical joint at hip and shoulder, one for each hinge joint at elbow and knee and two for the rotations in each ankle (van den Bogert *et al.* 1994). Positions and orientations of joint axes were based on existing methods (Isman and Inman 1969, Bell *et al.* 1990, Vaughan *et al.* 1992). Segment mass properties were computed from total body mass and segment lengths using the methods of de Leva (1996). Equations of motion were generated using SD/Fast as described in section 2.1. Data from the running trials were processed using the methods presented in sections 2.1–2.3. Specifically, the spline smoothing of $\mathbf{q}(t)$ and $\boldsymbol{\tau}_{\mathbf{k}}(t)$ was performed using a quintic spline filter (Woltring 1986) with a cutoff frequency of 8 Hz. After smoothing, force and motion variables were resampled at a frame rate of 240 Hz, starting at heel strike of each trial, for a total of 100 frames (417 ms). In each trial, the inverse dynamic analysis was performed three ways:

(1) Using full GRF data (FULLGRF). This represents the situation with 29 equations (one for each DOF) and 23 unknowns (one for each joint moment). The covariance matrix was generated from the assumption of 1 mm error in all marker coordinates and 0.1 N and 0.1 Nm error in GRF and moment data, respectively.
(2) Without using GRF measurements (NOGRF). This represents the situation with 29 equations and 29 unknowns (one for each joint moment, and the six unknown GRF variables). This is not an over-determined system and results are independent of the covariance matrix.
(3) Using partial GRF data, simulating the instrumented treadmill in which only the vertical force and center of pressure are measured (FzMxy). This represents the situation with 29 equations and 26 unknowns (joint moments and three unknown GRF variables). The covariance matrix was generated from the assumption of 50 mm error in all marker coordinates and 0.1 N and 0.1 Nm error in GRF and moment data.

Solution (1) is the best possible solution with all available data and will be used as the "gold standard". Solutions (2) and (3) represent two options for analysis of running on the partially instrumented treadmill. Solution (2) can be found with existing recursive inverse dynamic analysis, starting at the hands and working towards the lower extremity. Solution (3) requires our weighted least squares method.

The comparison between the three analyses will consider six variables of interest, the 3D joint moments at hip and knee which are thought to be relevant for overuse injury (Ferber *et al.* 2003). We will present the time histories of these variables during one typical trial, using all three solution methods. The ability of methods (2) and (3) to detect differences between trials was assessed by determining peak joint moments from each trial over the first 60 frames (250 ms) after heelstrike. Each of the methods (2) and (3) was compared to the "gold standard" result of method (1) and the differences between methods were quantified by the Pearson product-moment correlation coefficient

$$r = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}} \tag{9}$$

and mean relative error

$$\mathrm{MRE} = \frac{1}{n}\sum\left|\frac{x_i - y_i}{y_i}\right| \tag{10}$$

where x_i is the peak joint moment in trial i obtained with method X, and y_i is the corresponding "gold standard" value obtained with the FULLGRF method.

4. Results

Figure 2 shows, for one typical trial, the three dimensional joint moments at the hip and knee, computed with all three methods. There is good agreement between the three methods for the flexion–extension moments. The agreement appears to be worst for the internal–external rotation, especially in the swing phase.

Peak joint moments for all trials are shown in figures 3 and 4 and the corresponding quantitative comparisons are reported in table 1. The partial instrumentation (FzMxMy) results which were produced using the LSID technique correlated well with FULLGRF results, except for the hip extensor moment. The error, however, in the hip extensor moment was only 13.5%, suggesting that the low correlation in this variable is due to small variations between trials. The opposite is true for the knee rotator moment, which has a large error of 47.2% but a high correlation coefficient. This shows that the error is mostly systematic (figure 4, bottom right) and that increases or decreases in this variable can still be detected well with partial GRF data. The NOGRF method (figure 3) had larger errors and lower correlations when compared to FULLGRF, especially for the knee adductor moment. NOGRF also systematically overestimated the hip abductor moment and underestimated the knee extensor moment.

5. Discussion

We have presented a general methodology for performing inverse dynamic analysis of multibody systems, which makes optimal use of the redundancy in kinematic and external force data. Compared to earlier versions of this

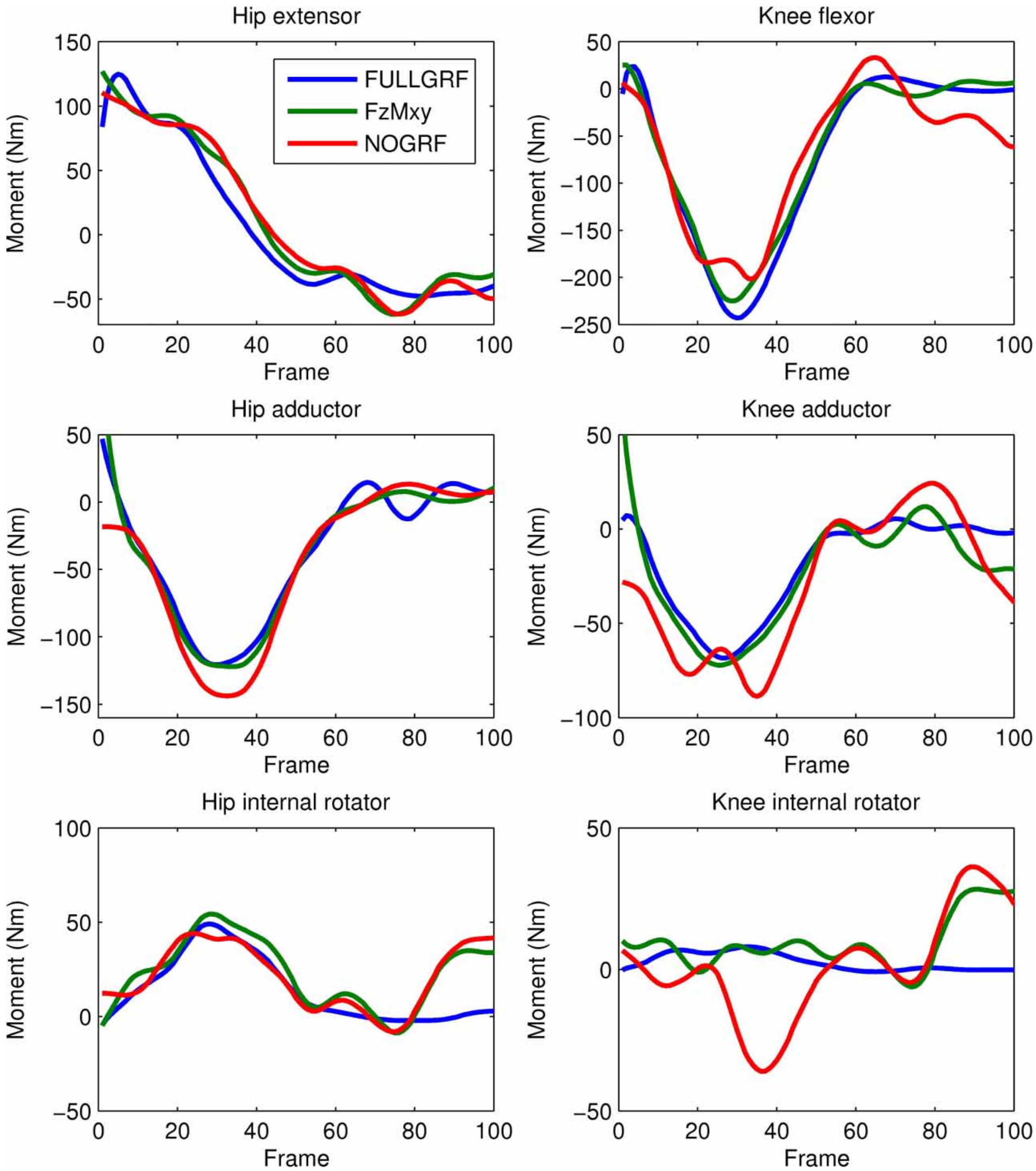

Figure 2. Three dimensional joint moments at hip and knee during a representative running trial, obtained with each of the three inverse dynamics methods. Frame rate is 240 Hz and frame 1 represents heel strike.

method (Kuo 1998), it is no longer required that the model is jointed to the ground. This, however, necessitated the use of a weighting matrix in solving the least squares problem. We derived this weighting matrix via Monte Carlo simulation of error propagation from estimated errors in raw measurements (marker trajectories and external forces). The method as presented here solves unknown actuator forces, joint moments and any unknown GRFs. Conventional recursive methods (Winter 1979) also solve the non-actuating reaction loads at each joint, which are useful in estimations of joint contact forces (van den Bogert 1994). In order to enable such applications of our methods, not demonstrated in this paper, we obtain the full 6-component reaction loads at each joint by a single function call to the SD/Fast function SDREAC, after the actuating loads have been solved. In addition to the ability to utilize redundant measurements, the least squares method has the additional advantage over conventional methods that it is not limited to tree-structured multibody systems.

There are also some limitations and disadvantages of this method. If certain regions of the multibody system have higher errors in model or measurements, it may be better not to use a whole body least squares method, but (if complete external force data are available) use a recursive method which models only the region of interest. For example, the inverse dynamic analysis of the lower extremity during gait is more reliable when the upper body

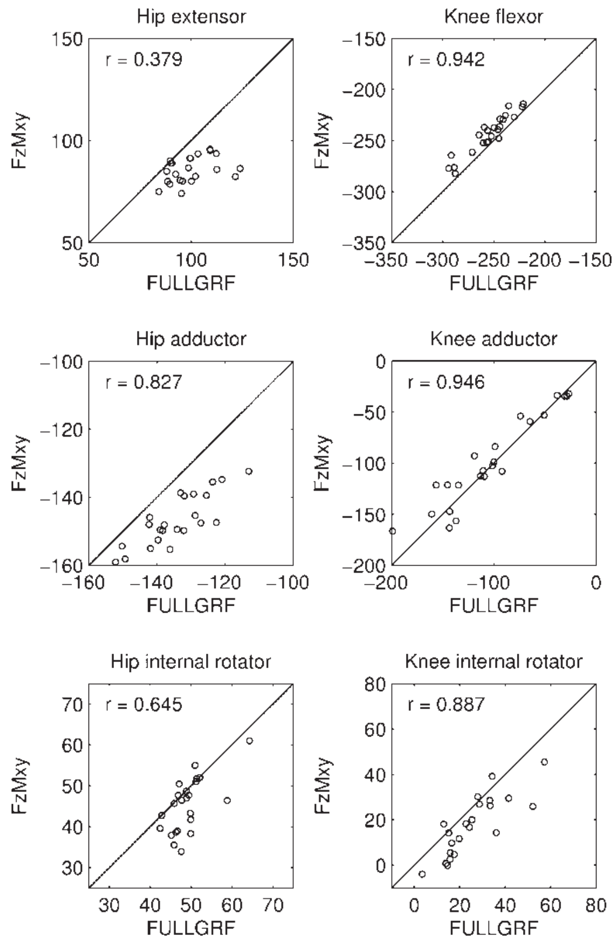

Figure 3. Peak joint moments (in Nm) in all 23 trials, compared between the NOGRF method (no GRF data used) and the FULLGRF method, where full GRF data is available. Maximum moments were used for hip extensor and both internal rotator moments. Minimum moments were used for knee flexor and both adductor moments.

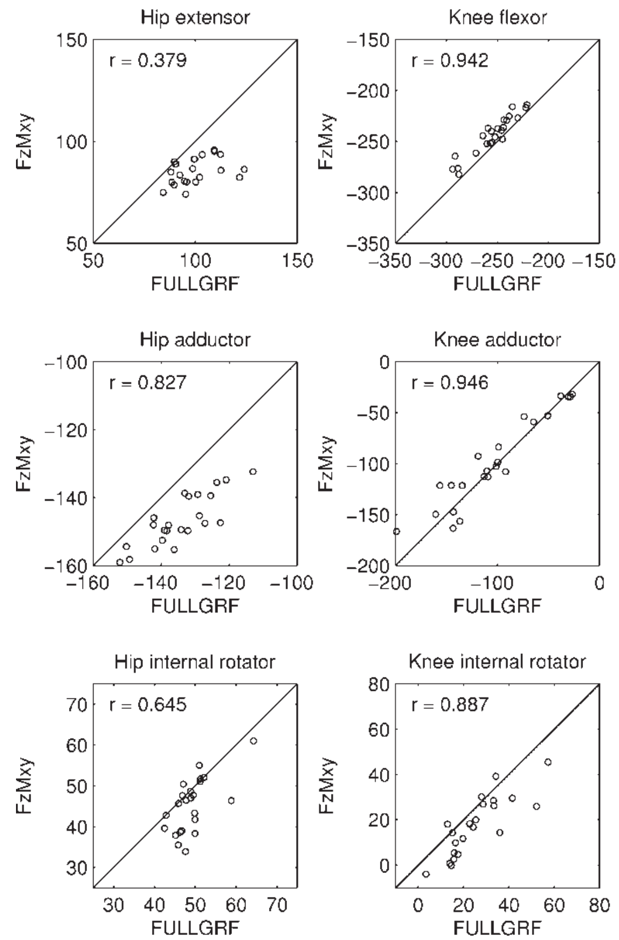

Figure 4. Peak joint moments in all 23 trials, compared between the FzMxy method (partial instrumentation) and the FULLGRF method, where full GRF data is available.

is not included in the model, because motion of visceral mass can not be measured reliably. However, if no full GRF data is available, such a partial body model is not an option.

The hip and knee joint moments during running (figure 2) were consistent with other studies that used full force plate instrumentation (Winter 1983, Ferber *et al.* 2003). Our results demonstrate that nearly the same results can be obtained with an instrumented treadmill in which only the vertical force and centre of pressure are measured (figure 4), except perhaps the extensor moment at the hip. Large errors were seen in the internal rotation moments during the swing phase (figure 2), when the true loads are zero, but the unmeasured GRF were given significantly non-zero estimates by the NOGRF and FzMxy methods. This problem could be avoided by considering these unmeasured variables to be known, and equal to zero, whenever the measured vertical GRF (Fz) is zero.

In order to obtain good results with the LSID method, the measurement error estimates σ_r and σ_τ, required for the covariance matrix, may need to be tuned carefully. In this application, the FULLGRF results were robust and not sensitive to our choice of $\sigma_r = 1\,\mathrm{mm}$ and $\sigma_\tau = 0.1\,\mathrm{N}$ and 0.1 Nm. However, the FzMxy analysis was sensitive to σ_r. Trial and error tuning showed that $\sigma_r = 50\,\mathrm{mm}$ produced good results, though no attempt was made to fully optimize this parameter. The value of 50 mm may seem large, as it is much larger than typical measuring errors in motion capture systems, but it does reflect the fact that many aspects of upper body motion, where most mass resides, were not modeled: spine and neck motion, scapulo-thoracic translation, wrist motion, and especially the motion of internal organs which can not be measured reliably.

Table 1. Error measures for joint moments obtained with no force measurement (NOGRF) or partial force measurement (FzMxMy). Each result was compared to a "gold standard" where complete GRF data was used (FULLGRF). Mean relative errors (MRE) and correlation coefficients (r) were computed using equations (9) and (10).

	FzMxMy		NOGRF	
Joint moment	MRE (%)	r	MRE (%)	r
Hip extensor	13.5	0.379	11.4	0.263
Hip adductor	9.4	0.827	16.9	0.735
Hip rotator	9.4	0.645	12.4	0.653
Knee flexor	4.4	0.942	12.8	0.663
Knee adductor	11.7	0.946	58.4	0.002
Knee rotator	47.3	0.887	49.6	0.526

We have demonstrated the utility of the LSID method for analysis of running on a partially instrumented treadmill. This can be applied clinically to assist and evaluate gait retraining therapies with the goal of preventing overuse injuries in runners. Many such injuries are thought to be related to abnormal three dimensional joint moments (Ferber *et al.* 2003). The partially instrumented analysis (FzMxMy) has the capability of detecting changes in joint moments caused by changes in running technique.

When analysing walking gait with similar methods, we obtained good results during single stance, but the double stance phase presents a problem, even if the feet are on separate force platforms that can measure vertical force and center of pressure. Although the number of unknowns (N-6 joint moments, plus 6 external force/moment variables) is, in this case, exactly equal to the number of equations N, the matrix $\mathbf{A}$ is singular and no unique solution exists. This can be understood by considering that the resultant horizontal GRF can be estimated from horizontal acceleration of the center of mass of the entire body, but the data contains no information on how this resultant force is distributed between the two feet. Minimal effort solutions can then be considered as an alternative (Vaughan *et al.* 1982). An application where the LSID method may be especially useful is the spine, where it will produce an optimal merging of the top–down and bottom–up methods which are currently the only available options (de Looze *et al.* 1992).

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